FINAL REPORT

COMPOSITE MATERIAL INTERFACE MECHANICS

GRANTS
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These grants were funded jointly by the Air Force Office of Scientific Research and by the Office of Naval Research through the National Center for Composite Materials at the University of Illinois, Urbana-Champaign. The effective time period of these grants was March 1, 1989 - September 30, 1991 with a no-cost extension for the period October 1 - December 31, 1991. The work of the principal investigator was done at the Dept. of Mechanical Engineering and Applied Mechanics, University of Pennsylvania.

The major part of the scientific work has been concerned with quantification of the concept of imperfect interface, the modifications of the theory of composite materials for imperfect interface and evaluation of the effect of various kinds of imperfect interface on various properties of various composite materials. In addition a study of the effect of matrix nonlinearity and imperfect interface on the critical compressive stress of unidirectional fiber composites has been performed. The work performed has been published and submitted for publication in 8 papers which are listed below.

1. Z.Hashin, "Thermoelastic properties of fiber composites with imperfect interface", Mechanics of Materials, **8**, 333-348, (1990).

2. Z. Hashin, "Composite materials with viscoelastic interphase", Mechanics of Materials, 11, 135-148, (1991)

3. Z. Hashin, "Composite materials with interphase : Thermoelastic and viscoelastic effects", in Inelastic Deformation of Composite Materials, G.J. Dvorak, Ed., Springer Verl., 3-34, (1991).

4. Z. Hashin, "Micromechanics aspects of damage in composite materials", in Durability of Composite Materials Systems, A.H. Cardon and G. Verchery, Eds.

Elsevier Applied Science, 27-45, (1991).

5. Z. Hashin, "The spherical inclusion with imperfect interface conditions", J. Appl. Mech., **58**, 444-449, (1991).

6. Z. Hashin, "Thermoelastic properties of particulate composites with imperfect interface", J.Mech.Physics Solids, **39**, 745-762, (1991)

7. Z. Hashin, "Extremum principles for elastic heterogeneous media with imperfect interface and their application to bounding of effective moduli", J.Mech.Physics Solids (in press).

8. Z. Hashin, "Effect of matrix nonlinearity and imperfect interface on compressive strength of fiber composites", (to be published)

Attached are copies of the first pages of the above listed publications, including abstracts.

In addition the principal investigator has given invited lectures on these subjects at the following places and occasions, during the grant period :

Yale University
University of North Carolina (Raleigh) - Distinguished Lecture Seminar
Princeton University
University of Pennsylvania
Arizona State University
Rensselaer Polytechnic Institute
University of Delaware
Free University of Brussels, Belgium
University of Illinois, Urbana
Annual Meeting of the Society of Engineering Science, Eringen Medal acceptance lecture, Ann Arbor, MI, October 1989.
IUTAM Symposium on Inelastic Behavior of Composite Materials, RPI, NY, August 1990
Annual Meeting of the Society of Engineering Science, Santa Fe, NM, 1990

Statement A per telecon Dr. R. Barsoum
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THERMOELASTIC PROPERTIES OF FIBER COMPOSITES WITH IMPERFECT INTERFACE

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Received 20 September 1989

Imperfect interface conditions are defined in terms of linear relations between interface tractions and displacement jumps. All of the thermoelastic properties of unidirectional fiber composites with such interface conditions are evaluated on the basis of the generalized self consistent scheme (GSCS) model. Results for elastic interphase are obtained as a special case by evaluation of interface parameters in terms of interphase characteristics. Numerical evaluation has shown that imperfect interface may have a significant effect on transverse thermal expansion coefficient, transverse shear and Young's moduli and axial shear modulus, a moderate effect on axial Poisson's ratio, small effect on axial thermal expansion coefficient and an insignificant effect on axial Young's modulus.

1. Introduction

The literature on the analytical determination of the thermoelastic properties of composite materials has almost in all cases been concerned with what will be here referred to as *perfect interfaces*, which means that tractions and displacements are continuous across the interface. However, in many cases of interest the perfect interface is not an adequate model. One such situation is the presence of a thin layer or coating, enveloping the reinforcing constituent. Such an interfacial layer is generally referred to as *interphase*. It may be due to chemical interaction between the constituents or it may be introduced by design in order to improve the properties of the composite. Recognition of the interphase as such implies that the composite is regarded as a three phase material. Such a description requires knowledge of the interphase properties, information which is rarely available primarily because the interphase material properties are in situ properties which are not necessarily equal to bulk properties.

An alternative model is based on the recognition that if the interphase is to have significant effect its properties must be significantly different from those of the constituents, in general much more flexible. Then the interphase effect may be interpreted as a displacement jump when traversing the interphase from reinforcement to matrix while the tractions must remain continuous from simple equilibrium considerations. In the simplest representation the jumps in normal and tangential displacement are proportional to the associated traction components. Thus referring to Fig. 1,

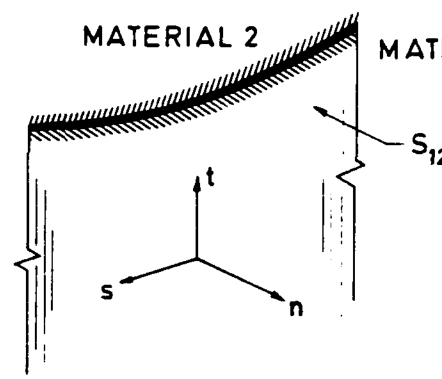

Fig. 1. Interface.

$$\sigma_{nn}^{(1)} = \sigma_{nn}^{(2)} = D_n[u_n], \qquad [u_n] = u_n^{(1)} - u_n^{(2)},$$
$$\sigma_{ns}^{(1)} = \sigma_{ns}^{(2)} = D_s[u_s], \qquad [u_s] = u_s^{(1)} - u_s^{(2)}, \qquad (1)$$
$$\sigma_{nt}^{(1)} = \sigma_{nt}^{(2)} = D_t[u_t], \qquad [u_t] = u_t^{(1)} - u_t^{(2)},$$

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Composite materials with viscoelastic interphase: creep and relaxation

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Received 8 May 1990; revised version received 16 August 1990

The effect of viscoelastic interphase between elastic constituents of composites is investigated on the basis of a correspondence principle relating effective elastic and viscoelastic properties of composites, which has been previously established. Previously derived results for thermoelastic properties of unidirectional fiber composites and spherical particle composites with elastic interphase are utilized in this fashion to obtain corresponding viscoelastic effective properties. The analysis shows that significant viscoelastic effect in such unidirectional composites is confined to axial and transverse shear relaxation and creep and in particulate composites to any shear loading. The viscoelastic effect is negligible for all inputs which do not include such shear components.

1. Introduction

In recent years there has been considerable interest in the effect of the nature of the interface on the properties of composite materials. Imperfect interface conditions have been quantitatively defined as linear relations between interface displacement discontinuities and associated traction components. The effect of such interfaces on the elastic properties of composite materials has been investigated in Benveniste (1985), for particulate composites, Achenbach and Zhu (1990), numerically, for fiber composites modeled as periodic hexagonal arrays, Hashin (1990a), for fiber composites, and Hashin (1990b) for particulate composites. The last two papers also dealt with the effect of interface on thermal expansion. In particular it was shown in Hashin (1990a, 1990b) that the effect of an interphase, by which is meant a thin coating on the reinforcing fibers or particles, can be expressed in terms of imperfect interface conditions and the interface parameters involved can be determined in terms of interphase characteristics.

In the present work we consider the case of viscoelastic interphase between two elastic components. An interphase of such nature may be a by-product of manufacture or it might be introduced by design, e.g. to provide relaxation and damping characteristics to an otherwise elastic brittle composite. It is our purpose to evaluate effective quasi-static viscoelastic properties of such composites. We do this on the basis of analytical results for elastic composites with interphase given in Hashin (1990a, 1990b), and correspondence principles for viscoelastic composites established in Hashin (1965, 1966).

The case of viscoelastic interphase in unidirectional fiber composites has also been recently considered by Gosz et al. (1990) in terms of finite element analysis of periodic hexagonal arrays and also some simple applications of the correspondence principle.

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Inelastic Deformation of Composite Materials

IUTAM Symposium, Troy, New York
May 29-June 1, 1990

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Composite Materials with Interphase: Thermoelastic and Inelastic Effects

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ABSTRACT

The effect of thin interphase between constituents of a composite material is described in terms of imperfect interface conditions which involve interface parameters. Elastic, viscoelastic and elastoplastic interphases are considered and their effect on the mechanical properties of composites is evaluated on the basis of the composite cylinder/spheres assemblage models and the generalized self consistent scheme approximation.

INTRODUCTION

The effective properties of a composite material depend on two kinds of information: the properties of the constituents and the

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DURABILITY OF POLYMER BASED COMPOSITE SYSTEMS FOR STRUCTURAL APPLICATIONS

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CONTENTS

MICROMECHANICS ASPECTS OF DAMAGE IN COMPOSITE MATERIALS

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ABSTRACT

This presentation is concerned with damage in composite materials in the form of many microcracks. Some general results for cracked homogeneous and composite bodies are reviewed. Theory of stiffness reduction and thermal expansion change of laminates due to crack accumulation is discussed. Recent theory of prediction of crack density in laminates is presented. The phenomenon of residual macrostrains after unloading of elastic brittle composites is explained and quantified.

INTRODUCTION

The response of composite materials to mechanical and/or thermal loads may be divided into two main regimes. In the first no significant internal changes take place and it is for this regime that the effective properties such as stiffness, thermal expansion coefficients (TEC) and conductivity are usually evaluated, assuming in particular perfect interface bonding. In the second regime internal defects accumulate. Such defects may be matrix cracks, fiber ruptures, fiber buckling and interface disbonds. The aggregate of such defects may be called damage and their global effect is to reduce effective properties. The accumulation of damage with load terminates in failure of the composite. It would appear that since the damage process is a prelude to failure, the analysis of damage should be a prerequisite to the prediction of failure.

Micromechanics implies mathematical analysis based on the principles of mechanics, recognizing the detailed geometry of the defects. This is of course a very difficult undertaking since problems of interacting defects are notoriously difficult even in homogeneous media, and in composites the difficulties are compounded by the presence of constituent interfaces.

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The Spherical Inclusion With Imperfect Interface

A rigorous solution of the spherical inclusion problem with remote uniform strain or stress and imperfect elastic spring-type interface conditions is presented. In the case of thin elastic interphase, the interface spring constants are expressed in terms of interphase elastic properties and thickness. The state of strain/stress in the inclusion is nonhomogeneous for imperfect interface conditions in contrast to the homogeneous state for perfect interface condition. Numerical results are given to demonstrate the significance of imperfect interface effects.

Introduction

Problems of the determination of stress fields in the vicinity of foreign inclusions embedded in a matrix material have been given considerable attention in the mechanics of solids literature, both in the stress concentration context and in relation to composite materials problems. Almost all of the work has been concerned with the classical situation when displacements and tractions are continuous across the inclusion matrix interface. This will here be referred to as perfect interface conditions, and within this context the primary achievement is the well-known Eshelby (1957) solution of the ellipsoidal inclusion problem.

Recently, there has been considerable interest in imperfect interface conditions as may be appropriate in the case of thin coatings on inclusions, produced by design or by chemical interaction, or for interface damage due, for example, to cyclic loading. Walpole (1978) has discussed the coated inclusion problem, in a general sense, on the basis of Hill's (1961, 1972) interface discontinuity theorems.

Mura et al. (1985) have given a solution for the axisymmetric case of a sliding spheroidal inclusion, assuming perfect normal interface bonding and ideally lubricated tangential contact. Ghahremani (1985) gave a solution for the more special case of a spherical inclusion under the same conditions. Mikata and Taya (1985) solved the axisymmetric case of a coated spheroidal inclusion which is embedded in an infinite medium. Using techniques to be described in this paper, this solution could be interpreted to apply for the case of imperfect normal and tangential imperfect interface conditions, thus generalizing the solution of Mura et al. Benveniste (1985) has treated an embedded composite sphere problem with perfect normal interface bonding and imperfect shear bonding for the case of remote shear. For literature on related problems of composite materials with coated reinforcement see, e.g., Hashin (1990).

The present work is concerned with solution of the spherical inclusion problem in the case of imperfect elastic normal and tangential bond between inclusion and matrix.

Formulation

Suppose that two different homogeneous media, which are labeled 1 and 2, are joined by an imperfect interface, S_{12} (Fig. 1). Then the quantitative effect of the imperfection is in the form of displacement discontinuities across the interface while the tractions must remain continuous for reasons of equilibrium. The simplest mathematical description of elastic imperfect interface is based on the assumption that normal and tangential displacement discontinuities are proportional to the respective traction components. Thus,

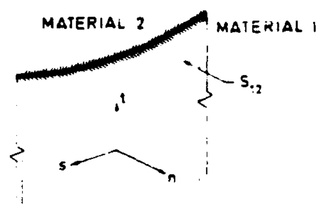

Fig. 1 Interface

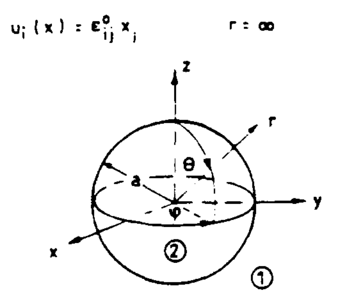

Fig. 2 Spherical inclusion

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THERMOELASTIC PROPERTIES OF PARTICULATE COMPOSITES WITH IMPERFECT INTERFACE

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(Received 28 March 1990)

ABSTRACT

EFFECTIVE elastic moduli and thermal expansion coefficient of spherical particle composites with imperfect interfaces are evaluated on the basis of the composite spheres assemblage and generalized self-consistent scheme models. Imperfect interface is defined in terms of interface displacement discontinuities which are linearly related to interface tractions in terms of spring constant parameters. In the case of presence of interphase these parameters are evaluated in terms of interphase characteristics.

INTRODUCTION

THIS PAPER is concerned with the evaluation of the effective elastic properties and the thermal expansion coefficient of composite materials consisting of isotropic matrix and randomly dispersed spherical particles when the interface bond is imperfect. Imperfect interface bond may be due to a very compliant thin interfacial layer, from now on referred to as interphase, or to interface damage. The effect of such interface imperfection on the properties and failure mechanisms of composite materials is of primary interest in composite materials research. As in previous papers on the subject, MAL and BOSE (1975), BENVENISTE (1985), ACHENBACH and ZHU (1989, 1990) and HASHIN (1990, 1991) we describe the imperfect interface in terms of interface displacement jumps which are linearly related to associated interface tractions, while interface tractions remain continuous for reasons of equilibrium. Thus with respect to a local orthogonal system of axes n, s, and t originating at some point on the interface, where n is the normal direction and s, t are tangential directions (Fig. 1), we describe imperfect interface conditions by the relations

$$T_n^{(1)} = T_n^{(2)} = D_n[u_n], \qquad [u_n] = u_n^{(1)} - u_n^{(2)}, \tag{1a}$$

$$T_s^{(1)} = T_s^{(2)} = D_s[u_s], \qquad [u_s] = u_s^{(1)} - u_s^{(2)}, \tag{1b}$$

$$T_t^{(1)} = T_t^{(2)} = D_t[u_t], \qquad [u_t] = u_t^{(1)} - u_t^{(2)}, \tag{1c}$$

where D_n, D_s, and D_t are spring constant type parameters which will be called interface

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EXTREMUM PRINCIPLES FOR ELASTIC HETEROGENOUS MEDIA WITH IMPERFECT INTERFACES AND THEIR APPLICATION TO BOUNDING OF EFFECTIVE MODULI

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Department of Mechanical Engineering and Applied Mechanics, University of Pennsylvania, Philadelphia, PA 19104-6315, U.S.A.

(Received 17 June 1991)

ABSTRACT

THE ELASTICITY extremum principles of minimum potential and minimum complementary energies are extended to the case of heterogenous media with imperfect interface conditions. The extended principles are applied to construct bounds for the effective elastic moduli of two-phase materials and polycrystalline aggregates with imperfect interfaces.

INTRODUCTION

THERE exists today a very large literature on the mechanics of composites or heterogenous media, and literally all of it is based on the assumption that the constituents are perfectly bonded together, which implies that displacements are continuous at the interface. In recent years there has been considerable interest in situations in which such an assumption is not appropriate. A simple mechanical model of an imperfect interface is described by the assumption that normal and tangential components of interface displacement jumps are proportional to the respective interface traction components in terms of spring-constant-type interface parameters. The effect of such imperfect interface conditions on effective elastic properties of composite materials has been considered in MAL and BOSE (1975), BENVENISTE (1985), CHATTERJEE and KIBLER (1979), ACHENBACH and ZHU (1989, 1990) and HASHIN (1990, 1991a, b). It has been pointed out in the work of the last author that the effect of thin flexible coating of the reinforcement, i.e. an interphase between the constituents, is equivalent to imperfect interface conditions of the type described above. It has moreover been shown that the interface parameters can be evaluated in terms of the interphase thickness and properties.

The effect of a thin plane interphase between generally anisotropic media has been discussed by BASSANI and QU (1990).

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EFFECT OF MATRIX NONLINEARITY AND IMPERFECT INTERFACE ON COMPRESSIVE STRENGTH OF FIBER COMPOSITES

by

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SUMMARY

Compressive strength of unidirectional fiber composites is identified with compressive stress in fiber direction which produces bifurcation of fiber axes, from now on called the critical stress. A continuum energy approach is adopted which is based on the effective properties of the fiber composite, thus avoiding the difficult problem of micromechanics of buckled fibers. Assuming that bifurcation occurs in a shear mode, fig. 1, it is shown that for linear elastic constituents the critical stress is equal to the axial effective shear modulus of the composite. This has been demonstrated in the literature for special two dimensional materials and has here been shown for the first time for **any** unidirectional fiber composite. Strictly speaking the result is an upper bound on the critical stress. It is, however, believed on physical grounds, that the shear mode is the actual bifurcation mode and experiments seem to support this.

In the case of nonlinear matrix it is shown by the same approach that the critical stress is given by the effective tangent shear modulus of the composite, at bifurcation. This defines a nonlinear equation for the critical stress. Using a Ramberg/Osgood J_2 presentation of the stress-strain relations of the matrix the critical stress has been obtained for 3501-6 Epoxy matrix and various kind of fibers. The results , together with corresponding critical stress results for the same matrix with initial elastic properties, are shown in table 1. It is seen that matrix nonlinearity produces significant reduction of linear critical stress, as has also been observed experimentally.

The present approach permits the incorporation of the effect of interface imperfection in terms of its effect on the effective shear modulus. This has been studied previously [Z.Hashin, "Thermoelastic properties of fiber composites with imperfect interface", Mechanics of Materials, **8**, 333-348, (1990)]. The equivalent effect of imperfect interface is to lower the fiber axial shear modulus. This is illustrated in fig.2 which shows the effect of diminishing fiber shear modulus, thus interface imperfection on the critical compressive stress for AS4/Epoxy composite of .65 fiber volume fraction, for linear and nonlinear matrix. It is seen that the critical stress is significantly reduced by interface imperfection.

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The literature on the analytical determination of the thermoelastic properties of composite materials has almost in all cases been concerned with what will be here referred to as *perfect interfaces*, which means that tractions and displacements are continuous across the interface. However, in many cases of interest the perfect interface is not an adequate model. One such situation is the presence of a thin layer or coating, enveloping the reinforcing constituent. Such an interfacial layer is generally referred to as *interphase*. It may be due to chemical interaction between the constituents or it may be introduced by design in order to improve the properties of the composite. Recognition of the interphase as such implies that the composite is regarded as a three phase material. Such a description requires knowledge of the interphase properties, information which is rarely available primarily because the interphase material properties are in situ properties which are not necessarily equal to bulk properties.

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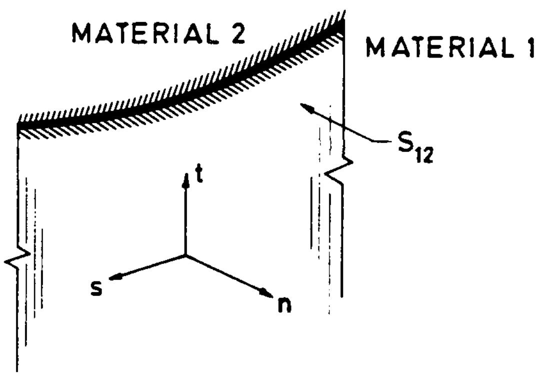

Fig. 1. Interface.

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Composite materials with viscoelastic interphase: creep and relaxation

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Received 8 May 1990; revised version received 16 August 1990

The effect of viscoelastic interphase between elastic constituents of composites is investigated on the basis of a correspondence principle relating effective elastic and viscoelastic properties of composites, which has been previously established. Previously derived results for thermoelastic properties of unidirectional fiber composites and spherical particle composites with elastic interphase are utilized in this fashion to obtain corresponding viscoelastic effective properties. The analysis shows that significant viscoelastic effect in such unidirectional composites is confined to axial and transverse shear relaxation and creep and in particulate composites to any shear loading. The viscoelastic effect is negligible for all inputs which do not include such shear components.

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CONTENTS

MICROMECHANICS ASPECTS OF DAMAGE IN COMPOSITE MATERIALS

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The Spherical Inclusion With Imperfect Interface

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The present work is concerned with solution of the spherical inclusion problem in the case of imperfect elastic normal and tangential bond between inclusion and matrix.

Formulation

Suppose that two different homogeneous media, which are labeled 1 and 2, are joined by an imperfect interface, S_{12} (Fig. 1). Then the quantitative effect of the imperfection is in the form of displacement discontinuities across the interface while the tractions must remain continuous for reasons of equilibrium. The simplest mathematical description of elastic imperfect interface is based on the assumption that normal and tangential displacement discontinuities are proportional to the respective traction components. Thus,

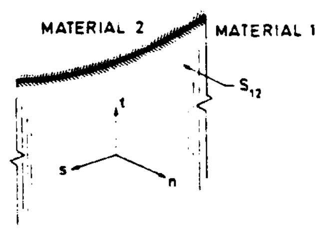

Fig. 1 Interface

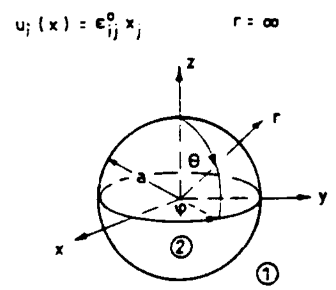

Fig. 2 Spherical inclusion

[1]On leave from Tel Aviv University, Tel Aviv, Israel.

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THERMOELASTIC PROPERTIES OF PARTICULATE COMPOSITES WITH IMPERFECT INTERFACE

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(Received 28 March 1990)

ABSTRACT

EFFECTIVE elastic moduli and thermal expansion coefficient of spherical particle composites with imperfect interfaces are evaluated on the basis of the composite spheres assemblage and generalized self-consistent scheme models. Imperfect interface is defined in terms of interface displacement discontinuities which are linearly related to interface tractions in terms of spring constant parameters. In the case of presence of interphase these parameters are evaluated in terms of interphase characteristics.

INTRODUCTION

THIS PAPER is concerned with the evaluation of the effective elastic properties and the thermal expansion coefficient of composite materials consisting of isotropic matrix and randomly dispersed spherical particles when the interface bond is imperfect. Imperfect interface bond may be due to a very compliant thin interfacial layer, from now on referred to as interphase, or to interface damage. The effect of such interface imperfection on the properties and failure mechanisms of composite materials is of primary interest in composite materials research. As in previous papers on the subject, MAL and BOSE (1975), BENVENISTE (1985), ACHENBACH and ZHU (1989, 1990) and HASHIN (1990, 1991) we describe the imperfect interface in terms of interface displacement jumps which are linearly related to associated interface tractions, while interface tractions remain continuous for reasons of equilibrium. Thus with respect to a local orthogonal system of axes n, s, and t originating at some point on the interface, where n is the normal direction and s, t are tangential directions (Fig. 1), we describe imperfect interface conditions by the relations

$$T_n^{(1)} = T_n^{(2)} = D_n[u_n], \qquad [u_n] = u_n^{(1)} - u_n^{(2)}, \tag{1a}$$

$$T_s^{(1)} = T_s^{(2)} = D_s[u_s], \qquad [u_s] = u_s^{(1)} - u_s^{(2)}, \tag{1b}$$

$$T_t^{(1)} = T_t^{(2)} = D_t[u_t], \qquad [u_t] = u_t^{(1)} - u_t^{(2)}, \tag{1c}$$

where D_n, D_s, and D_t are spring constant type parameters which will be called interface

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EXTREMUM PRINCIPLES FOR ELASTIC HETEROGENOUS MEDIA WITH IMPERFECT INTERFACES AND THEIR APPLICATION TO BOUNDING OF EFFECTIVE MODULI

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(*Received* 17 *June* 1991)

ABSTRACT

THE ELASTICITY extremum principles of minimum potential and minimum complementary energies are extended to the case of heterogenous media with imperfect interface conditions. The extended principles are applied to construct bounds for the effective elastic moduli of two-phase materials and polycrystalline aggregates with imperfect interfaces.

INTRODUCTION

THERE exists today a very large literature on the mechanics of composites or heterogenous media, and literally all of it is based on the assumption that the constituents are perfectly bonded together, which implies that displacements are continuous at the interface. In recent years there has been considerable interest in situations in which such an assumption is not appropriate. A simple mechanical model of an imperfect interface is described by the assumption that normal and tangential components of interface displacement jumps are proportional to the respective interface traction components in terms of spring-constant-type interface parameters. The effect of such imperfect interface conditions on effective elastic properties of composite materials has been considered in MAL and BOSE (1975), BENVENISTE (1985), CHATTERJEE and KIBLER (1979), ACHENBACH and ZHU (1989, 1990) and HASHIN (1990, 1991a, b). It has been pointed out in the work of the last author that the effect of thin flexible coating of the reinforcement, i.e. an interphase between the constituents, is equivalent to imperfect interface conditions of the type described above. It has moreover been shown that the interface parameters can be evaluated in terms of the interphase thickness and properties.

The effect of a thin plane interphase between generally anisotropic media has been discussed by BASSANI and QU (1990).

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EFFECT OF MATRIX NONLINEARITY AND IMPERFECT INTERFACE ON COMPRESSIVE STRENGTH OF FIBER COMPOSITES

by

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SUMMARY

Compressive strength of unidirectional fiber composites is identified with compressive stress in fiber direction which produces bifurcation of fiber axes, from now on called the critical stress. A continuum energy approach is adopted which is based on the effective properties of the fiber composite, thus avoiding the difficult problem of micromechanics of buckled fibers. Assuming that bifurcation occurs in a shear mode, fig. 1, it is shown that for linear elastic constituents the critical stress is equal to the axial effective shear modulus of the composite. This has been demonstrated in the literature for special two dimensional materials and has here been shown for the first time for **any** unidirectional fiber composite. Strictly speaking the result is an upper bound on the critical stress. It is, however, believed on physical grounds, that the shear mode is the actual bifurcation mode and experiments seem to support this.

In the case of nonlinear matrix it is shown by the same approach that the critical stress is given by the effective tangent shear modulus of the composite, at bifurcation. This defines a nonlinear equation for the critical stress. Using a Ramberg/Osgood J_2 presentation of the stress-strain relations of the matrix the critical stress has been obtained for 3501-6 Epoxy matrix and various kind of fibers. The results , together with corresponding critical stress results for the same matrix with initial elastic properties, are shown in table 1. It is seen that matrix nonlinearity produces significant reduction of linear critical stress, as has also been observed experimentally.

The present approach permits the incorporation of the effect of interface imperfection in terms of its effect on the effective shear modulus. This has been studied previously [Z.Hashin, "Thermoelastic properties of fiber composites with imperfect interface", Mechanics of Materials, **8**, 333-348, (1990)]. The equivalent effect of imperfect interface is to lower the fiber axial shear modulus. This is illustrated in fig.2 which shows the effect of diminishing fiber shear modulus, thus interface imperfection on the critical compressive stress for AS4/Epoxy composite of .65 fiber volume fraction, for linear and nonlinear matrix. It is seen that the critical stress is significantly reduced by interface imperfection.

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THERMOELASTIC PROPERTIES OF FIBER COMPOSITES WITH IMPERFECT INTERFACE

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Received 20 September 1989

Imperfect interface conditions are defined in terms of linear relations between interface tractions and displacement jumps. All of the thermoelastic properties of unidirectional fiber composites with such interface conditions are evaluated on the basis of the generalized self consistent scheme (GSCS) model. Results for elastic interphase are obtained as a special case by evaluation of interface parameters in terms of interphase characteristics. Numerical evaluation has shown that imperfect interface may have a significant effect on transverse thermal expansion coefficient, transverse shear and Young's moduli and axial shear modulus, a moderate effect on axial Poisson's ratio, small effect on axial thermal expansion coefficient and an insignificant effect on axial Young's modulus.

1. Introduction

The literature on the analytical determination of the thermoelastic properties of composite materials has almost in all cases been concerned with what will be here referred to as *perfect interfaces*, which means that tractions and displacements are continuous across the interface. However, in many cases of interest the perfect interface is not an adequate model. One such situation is the presence of a thin layer or coating, enveloping the reinforcing constituent. Such an interfacial layer is generally referred to as *interphase*. It may be due to chemical interaction between the constituents or it may be introduced by design in order to improve the properties of the composite. Recognition of the interphase as such implies that the composite is regarded as a three phase material. Such a description requires knowledge of the interphase properties, information which is rarely available primarily because the interphase material properties are in situ properties which are not necessarily equal to bulk properties.

An alternative model is based on the recognition that if the interphase is to have significant effect its properties must be significantly different from those of the constituents, in general much more flexible. Then the interphase effect may be interpreted as a displacement jump when traversing the interphase from reinforcement to matrix while the tractions must remain continuous from simple equilibrium considerations. In the simplest representation the jumps in normal and tangential displacement are proportional to the associated traction components. Thus referring to Fig. 1,

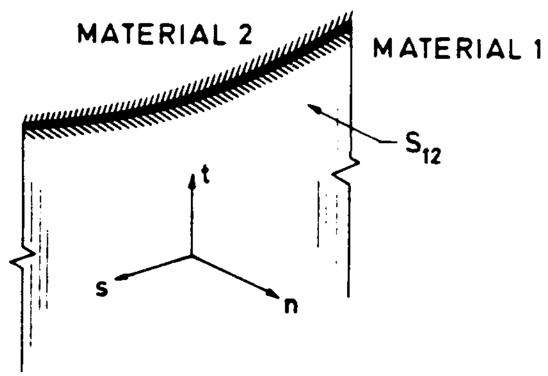

Fig. 1. Interface.

$$\begin{aligned}
\sigma_{nn}^{(1)} &= \sigma_{nn}^{(2)} = D_n[u_n], & [u_n] &= u_n^{(1)} - u_n^{(2)},\\
\sigma_{ns}^{(1)} &= \sigma_{ns}^{(2)} = D_s[u_s], & [u_s] &= u_s^{(1)} - u_s^{(2)}, \qquad (1)\\
\sigma_{nt}^{(1)} &= \sigma_{nt}^{(2)} = D_t[u_t], & [u_t] &= u_t^{(1)} - u_t^{(2)},
\end{aligned}$$

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Composite materials with viscoelastic interphase: creep and relaxation

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The effect of viscoelastic interphase between elastic constituents of composites is investigated on the basis of a correspondence principle relating effective elastic and viscoelastic properties of composites, which has been previously established. Previously derived results for thermoelastic properties of unidirectional fiber composites and spherical particle composites with elastic interphase are utilized in this fashion to obtain corresponding viscoelastic effective properties. The analysis shows that significant viscoelastic effect in such unidirectional composites is confined to axial and transverse shear relaxation and creep and in particulate composites to any shear loading. The viscoelastic effect is negligible for all inputs which do not include such shear components.

1. Introduction

In recent years there has been considerable interest in the effect of the nature of the interface on the properties of composite materials. Imperfect interface conditions have been quantitatively defined as linear relations between interface displacement discontinuities and associated traction components. The effect of such interfaces on the elastic properties of composite materials has been investigated in Benveniste (1985), for particulate composites. Achenbach and Zhu (1990), numerically, for fiber composites modeled as periodic hexagonal arrays. Hashin (1990a), for fiber composites, and Hashin (1990b) for particulate composites. The last two papers also dealt with the effect of interface on thermal expansion. In particular it was shown in Hashin (1990a, 1990b) that the effect of an interphase, by which is meant a thin coating on the reinforcing fibers or particles, can be expressed in terms of imperfect interface conditions and the interface parameters involved can be determined in terms of interphase characteristics.

In the present work we consider the case of viscoelastic interphase between two elastic components. An interphase of such nature may be a by-product of manufacture or it might be introduced by design, e.g. to provide relaxation and damping characteristics to an otherwise elastic brittle composite. It is our purpose to evaluate effective quasi-static viscoelastic properties of such composites. We do this on the basis of analytical results for elastic composites with interphase given in Hashin (1990a, 1990b), and correspondence principles for viscoelastic composites established in Hashin (1965, 1966).

The case of viscoelastic interphase in unidirectional fiber composites has also been recently considered by Gosz et al. (1990) in terms of finite element analysis of periodic hexagonal arrays and also some simple applications of the correspondence principle.

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Composite Materials with Interphase: Thermoelastic and Inelastic Effects

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ABSTRACT

The effect of thin interphase between constituents of a composite material is described in terms of imperfect interface conditions which involve interface parameters. Elastic, viscoelastic and elastoplastic interphases are considered and their effect on the mechanical properties of composites is evaluated on the basis of the composite cylinder/spheres assemblage models and the generalized self consistent scheme approximation.

INTRODUCTION

The effective properties of a composite material depend on two kinds of information: the properties of the constituents and the

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DURABILITY OF POLYMER BASED COMPOSITE SYSTEMS FOR STRUCTURAL APPLICATIONS

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CONTENTS

MICROMECHANICS ASPECTS OF DAMAGE IN COMPOSITE MATERIALS

Zvi Hashin[1]
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Tel Aviv, Israel

ABSTRACT

This presentation is concerned with damage in composite materials in the form of many microcracks. Some general results for cracked homogeneous and composite bodies are reviewed. Theory of stiffness reduction and thermal expansion change of laminates due to crack accumulation is discussed. Recent theory of prediction of crack density in laminates is presented. The phenomenon of residual macrostrains after unloading of elastic brittle composites is explained and quantified.

INTRODUCTION

The response of composite materials to mechanical and/or thermal loads may be divided into two main regimes. In the first no significant internal changes take place and it is for this regime that the effective properties such as stiffness, thermal expansion coefficients (TEC) and conductivity are usually evaluated, assuming in particular perfect interface bonding. In the second regime internal defects accumulate. Such defects may be matrix cracks, fiber ruptures, fiber buckling and interface disbonds. The aggregate of such defects may be called damage and their global effect is to reduce effective properties. The accumulation of damage with load terminates in failure of the composite. It would appear that since the damage process is a prelude to failure, the analysis of damage should be a prerequisite to the prediction of failure.

Micromechanics implies mathematical analysis based on the principles of mechanics, recognizing the detailed geometry of the defects. This is of course a very difficult undertaking since problems of interacting defects are notoriously difficult even in homogeneous media, and in composites the difficulties are compounded by the presence of constituent interfaces.

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The Spherical Inclusion With Imperfect Interface

A rigorous solution of the spherical inclusion problem with remote uniform strain or stress and imperfect elastic spring-type interface conditions is presented. In the case of thin elastic interphase, the interface spring constants are expressed in terms of interphase elastic properties and thickness. The state of strain/stress in the inclusion is nonhomogeneous for imperfect interface conditions in contrast to the homogeneous state for perfect interface condition. Numerical results are given to demonstrate the significance of imperfect interface effects.

Introduction

Problems of the determination of stress fields in the vicinity of foreign inclusions embedded in a matrix material have been given considerable attention in the mechanics of solids literature, both in the stress concentration context and in relation to composite materials problems. Almost all of the work has been concerned with the classical situation when displacements and tractions are continuous across the inclusion matrix interface. This will here be referred to as perfect interface conditions, and within this context the primary achievement is the well-known Eshelby (1957) solution of the ellipsoidal inclusion problem.

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Suppose that two different homogeneous media, which are labeled 1 and 2, are joined by an imperfect interface, S_{12} (Fig. 1). Then the quantitative effect of the imperfection is in the form of displacement discontinuities across the interface while the tractions must remain continuous for reasons of equilibrium. The simplest mathematical description of elastic imperfect interface is based on the assumption that normal and tangential displacement discontinuities are proportional to the respective traction components. Thus,

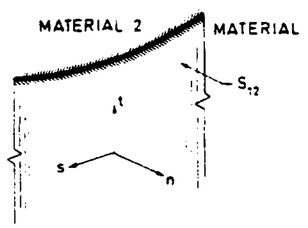

Fig. 1 Interface

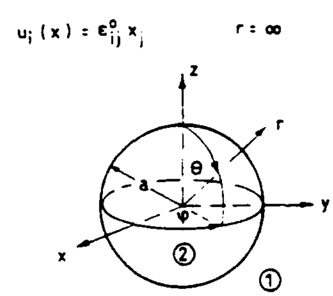

Fig. 2 Spherical inclusion

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EXTREMUM PRINCIPLES FOR ELASTIC HETEROGENOUS MEDIA WITH IMPERFECT INTERFACES AND THEIR APPLICATION TO BOUNDING OF EFFECTIVE MODULI

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by

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